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TUNNELLING TIMES AND THE "HARTMAN EFFECT"(*)

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Abstract

In a recent review paper [*Phys. Reports* **214** (1992) 339] we proposed, within conventional quantum mechanics, new definitions for the sub-barrier tunnelling and reflection times. Aims of the present paper are: (i) presenting and analysing the results of various numerical calculations (based on our equations) on the penetration and return times $\langle\tau_{Pen}\rangle$, $\langle\tau_{Ret}\rangle$, during tunnelling *inside* a rectangular potential barrier, for various penetration depths x_f; (ii) putting forth and discussing suitable definitions. besides of the mean values, also of the *variances* (or dispersions) $D\tau_T$ and $D\tau_R$ for the time durations of transmission and reflection processes; (iii) mentioning that our definition $\langle\tau_T\rangle$ for the average transmission time results to constitute an *improvement* of the ordinary dwell-time $\bar{\tau}^{DW}$ formula: (iv) commenting upon some recent criticism by C. R. Leavens, on the basis of our *new* numerical results. We stress that our numerical evaluations *confirm* that our approach implied, and implies, the existence of the *Hartman* effect: an effect that in these days (due to the theoretical connections between tunnelling and evanescent-wave propagation) is receiving – at Cologne, Berkeley, Florence and Wien – indirect, but quite interesting, experimental verifications. At last we briefly analyze some other definitions of tunnelling times.

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1. Introduction.

In our review article [1] [Phys. Rep. **214** (1992) 339] we put forth an analysis of the main theoretical definitions of the sub-barrier tunnelling and reflection times, and proposed new definitions for such durations which seem to be self-consistent within *conventional* quantum mechanics. This research field, however, is developing so rapidly, and in such a controversial manner, that during the last three years several new papers already did appear, which demand some more critical comment.(1) Moreover, the "prediction" by our theory [1] of the reality of the *Hartman effect* [2] in tunnelling processes has recently received – due to the analogy [3] between tunnelling electrons and evanescent waves – quite interesting, even if indirect, experimental verifications at Cologne, [4] Berkeley, [5] Florence [6] and Wien. [6]

(i) First of all, let us mention that we had overlooked a new expression for the dwell-time $\bar{\tau}^{Dw}$ derived by Jaworsky and Wardlaw [7,8]

$$bar\tau^{Dw}(x_i, x_f; k) = \left(\int_{-\infty}^{\infty} dtt\, J(x_f, t) - \int_{-\infty}^{\infty} dtt\, J(x_i, t) \right) \left(\int_{-\infty}^{\infty} dt\, J_{in}(x_i, t) \right)^{-1}, \tag{1}$$

which is indeed equivalent [7] to our eq. (16) of ref. [1] (all

(1) Let us take advantage of the present opportunity for pointing out that a missprint entered our eq. (10) in ref. [1], whose last term ka ought to be eliminated. Moreover, due to an editorial error, in the footnote at page 32 of our ref. [12] the dependence of G on Δk disappeared, whilst in that paper we had assumed $G(k - \bar{k}) \equiv C \exp[-(k - \bar{k})2/(\Delta k)^2]$.

notations being defined therein):

$$\bar{\tau}^{Dw}(x_i, x_f : f) = \left(\int_{-\infty}^{\infty} dt \int_{x_i}^{x_f} dx \rho(x,t)\right) \left(\int_{-\infty}^{\infty} dt J_{in}(x_i,t)\right)^{-1}. \tag{2}$$

This equivalence *reduces* the difference, between our definition $\langle\tau_T\rangle$ of the average transmission time – under our assumptions – and quantity $\bar{\tau}^{Dw}$, to the difference between the average made by using the positive-defined probability density $dt J_+(x,t) / \int_{-\infty}^{\infty} dt J_+(x,t)$ and the average made by using the ordinary "probability density" $dt J(x,t) / \int_{-\infty}^{\infty} dt J(x,t)$. Generally speaking, the last expression is *not* always positive defined, as it was explained at page 350 of ref. [1], and hence does not possess any direct physical meaning.

(ii) In ref. [9] an attempt was made to analyze the evolution of the wave packet mean position $\langle x(t)\rangle$ ("center of gravity"), averaged over ρdx, during its tunnelling through a potential barrier. Let us here observe that the conclusion to be found therein, about the absence of a causal relation between the incident space centroid and its transmitted equivalent, holds *only* when it is negligible the contribution to the space integral coming from the barrier region.

(iii) Let us also add that in ref. [10] it was analyzed the *distribution* of the transmission time τ_T in a rather sophisticated way, which is very similar to the dwell-time approach, however with an *artificial*, abrupt switching on of the initial wave packet. We are going to propose, on the contrary, and in analogy with our eqs. (30)-(31) in ref. [1], the following expressions, as physically adeguate definitions for the *variances* (or dispersions) $D\tau_T$

and $D\tau_R$ of the transmission and reflection time [see Sect. **2**], respectively:

$$D\tau_T \equiv Dt_+(x_f) + Dt_+(x_i) \tag{3}$$

and

$$D\tau_R \equiv Dt_-(x_i) + Dt_+(x_i), \tag{4}$$

where

$$Dt_\pm(x) \equiv \frac{\int_{-\infty}^{\infty} dt t^2 J_\pm(x,t)}{\int_{-\infty}^{\infty} dt J_\pm(x,y)} - \left(\frac{\int_{-\infty}^{\infty} dt t J_\pm(x,t)}{\int_{-\infty}^{\infty} dt J_\pm(x,y)} \right)^2 . \tag{5}$$

Equations (3)-(5) are based on the formalism expounded in ref. [11], as well as on our definitions for $J_\pm(x,t)$ in ref. [1]. Of course, we are supposing that the integrations over $J_+(x_f)dt$, $J_+(x_i)dt$ and $J_-(x_i)dt$ are independent of one another. We shall devote Sect. **2**, below, to these problems, i.e., to the problem of suitably defining mean values and variances of durations, for various transmission and reflection processes during tunnelling.

(iv) The results of our numerical calculations on the penetration and return durations *inside* a rectangular potential barrier during tunnelling will be presented and analysed in Sect. **3**; while those for the variances will appear elsewhere. Such evaluations seem to confirm that our approach is physically acceptable, and that it moreover implied, and implies, the existence of the so-called "*Hartman effect*" (even for *non*-quasi-monochromatic packets).

In so doing, we shall complete our comments on a paper by C. R. Leavens, [8] which criticized eqs. (30)-(31) of ref. [1] on the basis of various numerical calculations for the average transmission times. We had preliminary answered those criticisms in ref. [12]; but, subsequently, we discovered a misprint in the computer programme used by our group in Kiev, so that Figs. 1-3 of ref. [12] have to be modified: see Figs. 1-2 below. We shall see, however, that a disagreement still persists between Leavens' results and ours: even if some features of our new plots are more similar to Leavens' (and this is a wellcome step towards the solution of this problem). Such a disagreement can be merely due to the fact – as recently claimed also by Delgado et al. [13] – that *different initial* conditions for the wave packets were actually chosen in ref. [8] and in ref. [1]. Anyway, our approach seems to get support, at least in some particular cases, also by a recent article by Brouard et al., which generalizes – even if starting from a totally different point of view – some of our results [14,15].

We shall also answer Leavens' criticism on our analysis [1] of the dwell-time approaches [16].

(v) At last, we shall briefly re-analyse some other definition of tunnelling durations.

Before going on, let us recall that several reasons "justify" the existence of different approaches to the definition of tunnelling times: (a) the problem of defining tunnelling durations is closely connected with that of defining a time operator, i.e., of introducing *time* as a (non-selfadjoint) quantum mechanical observable, and subsequently of adopting a general definition for collision durations in quantum mechanics. Such preliminary problems did receive some clarification in recent times (see, for example. ref. [1] and citations [8] and [22] therein); (b) the motion of a particle tunnelling inside a potential barrier is a purely quantum phenomenon, devoid of any classical, intuitive limit; (c) the various theoretical approaches may differ in the choice of the boundary conditions or in the modelling of the experimental situations.

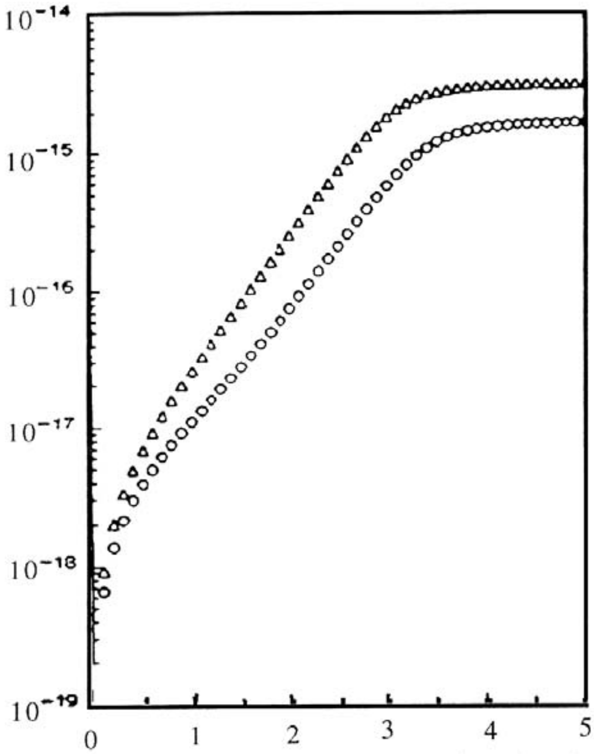

Fig. 1 - Behaviour of the average penetration time $\langle \tau_{Pen} \rangle$ (expressed in seconds) as a function of the penetration depth x_f (expressed in ångstroms) through a rectangular barrier with width $a = 5$ Å, for $\Delta k = 0.02$ Å^{-1} (small circles) and $\Delta k = 0.01$ Å^{-1} (small triangles), respectively. The other parameters are listed in footnote 2. It is worthwhile to notice that $\langle \tau_{Pen} \rangle$ rapidly increases for the first, few initial ångstroms (~ 2.5 Å), tending afterwards to a saturation value. This seems to confirm the existence of the so-called "Hartman effect". 14, 15

2. Mean values and Variances for various Penetration and Return Times during tunnelling.

In our previous papers, we proposed for the *transmission* and *reflection* times some formulae which imply – as functions of the penetration depth – integrations over time of $J_+(x,t)$ and $J_-(x,t)$, respectively. Let us recall that the total flux $J(x,t)$ in-

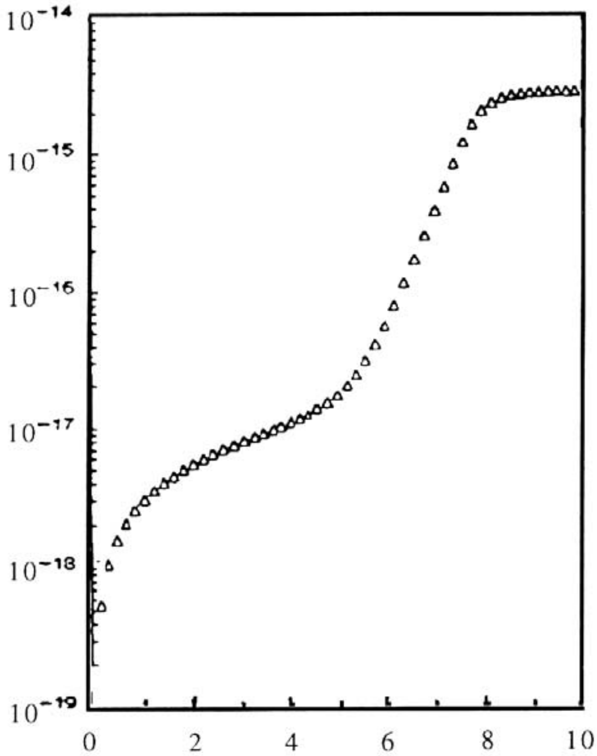

Fig. 2 - The same plot as in Fig. 1, for $\Delta k = 0.01$ Å^{-1}, except that now the barrier width is $a = 10$ Å. Let us observe that the numerical values of the (total) tunnelling time $\langle \tau_T \rangle$ practically does not change when passing from $a = 5$ Åto $a = 10$ Å, again in agreement with the characteristic features[1] of the Hartman effect. Figures 1 and 2 do improve (and correct) the corresponding ones, preliminarily presented by us in ref. [12].

side a barrier consists of two components, J_+ and J_-, associated with motion along the positive and the negative x-direction, respectively. Work in similar directions did recently appear in [14].

Let us, then, refer ourselves – here – to tunnelling and reflection processes of a particle by a potential barrier, confining, ourselves to one space dimension. Namely, let us study the evolution of a wave packet $\psi(x, t)$, starting from the initial state $\psi_{in}(x,t)$; and follow the notation introduced in ref. [1]. In the case of uni-directional motions it is already known [17] that the flux density $J(X,t) \equiv Re[(i\hbar/m)\psi(x,t)\partial\psi^*(x,t)/\partial x]$ can be actually interpreted as the probability that the particle (wave packet) passes through position x during a unitary time-interval centered at t, *as it easily follows from the continuity equation and from the fact that quantity* $\rho(x,t) \equiv |\psi(x,t)|^2$ *is the probability density* for our "particle" to be located, at time t, inside a unitary space-interval centered at x. Thus, in order to determine the *mean* instant at which a moving wave packet $\psi(x,t)$ passes through position x, we have to take the average of the time variable t with respect to the weight $w(x,t) \equiv J(x,t)/\int_{-\infty}^{\infty} J(x,t)dt$.

However, if the motion direction can *vary*, then quantity $w(x,t)$ is no longer positive defined, and moreover is not bounded, because of the variability of the $J(x,t)$ sign. In such a case, one can introduce the two weights:

$$w_+(x,t) = J_+(x,t)\left[\int_{-\infty}^{\infty} J_+(x,t)\right] dt \tag{6}$$

$$w_-(x,t) = J_-(x,t)\left[\int_{-\infty}^{\infty} J_-(x,t)\right] dt \tag{7}$$

where $J_+(x,t)$ and $J_-(x,t)$ represent the positive and negative parts of $J(x,t)$, respectively, which are bounded, positive-defined functions, normalized to 1. Let us show that, from the ordinary probabilistic interpretation of $\rho(x,t)$ and from the well-known continuity equation

$$\frac{\partial\rho(x,t)}{\partial t} + \frac{\partial J(x,t)}{\partial x} = 0, \tag{8}$$

it follows *also in this (more general) case* that quantities w_+ and w_-, represented by eqs. (6), (7), can be regarded as the probabilities that our "particle" passes through position x during a unit time-interval centered at t (in the case of forward and backward motion, respectively). Actually, for those time intervals for which $J = J_+$ or $J = J_-$, one can rewrite eq. (8) as follows:

$$(9.a) \qquad \frac{\partial \rho_>(x,t)}{\partial t} = -\frac{\partial J_+(x,t)}{\partial x}$$

$$(9.b) \qquad \frac{\partial \rho_<(x,t)}{\partial t} = -\frac{\partial J_-(x,t)}{\partial x}$$

respectively. Relations (9.a) and (9.b) can be considered as formal definitions of $\partial\rho_p > /\partial t$ and $\partial\rho_</\partial t$, whose physical meaning is going to be illustrated below. Let us integrate eqs. (9.a), (9.b) over time from $-\infty$ to t; we obtain:

$$(10.a) \qquad \rho_>(x,t) = -\int_{-\infty}^{t} \frac{\partial J_+(x,t')}{\partial x} dt'$$

$$(10.b) \qquad \rho_<(x,t) = -\int_{-\infty}^{t} \frac{\partial J_-(x,t')}{\partial x} dt'$$

with the initial conditions $\rho_>(x,-\infty) = \rho_<(x,-\infty) = 0$. Then, let us introduce the quantities

$$(11.a) \qquad N_>(x,\infty;t) \equiv \int_x^{\infty} \rho_>(x't)dx' = \int_{-\infty}^{t} J_+(x,t')dt' > 0$$

$$(11.b) \qquad \begin{aligned} N_<(-\infty,x;t) &\equiv \int_{-\infty}^{x} \rho_<(x',t)dx' \\ &= -\int_{-\infty}^{t} J_-(x,t')dt' > 0, \end{aligned}$$

which have the meaning of probabilities for our "particle" to be located at time t on the semi-axis (x, ∞) or $(-\infty, x)$ respectively, as functions of the flux densities $J_+(x, t)$ or $J_-(x, t)$, provided that the normalization condition $\int_{-\infty}^{\infty} \rho(x, t) dx = 1$ is fulfilled. The r.h.s.'s of eqs. (11.a) and (11.b) have been obtained by integrating the r.h.s.'s of eqs. (10.a) and (10.b) and adopting the boundary conditions $J_+(-\infty, t) = J_-(-\infty, t) = 0$. Now, differentiating eqs. (11.a) and (11.b) with respect to t, one obtains:

$$\frac{\partial N_>(x, \infty, t)}{\partial t} = J_+(x, t) > 0 \tag{12.a}$$

$$\frac{\partial N_<(x, -\infty, t)}{\partial t} = -J_-(x, t) > 0. \tag{12.b}$$

Finally, from eqs. (11.a), (11.b), (12.a) and (12.b), one can infer that:

$$w_+(x, t) = \frac{\partial N_>(x, \infty; t)/\partial t}{N_>(x, -\infty, \infty)} \tag{13.a}$$

$$w_-(x, t) = \frac{\partial N_<(x, -\infty; t)/\partial t}{N_<(-\infty, x; \infty)}, \tag{13.b}$$

which justify the abovementioned probabilistic interpretation of $w_+(x, t)$ and $w_-(x, t)$. Let us notice, incidentally, that our approach does *not* assume any ad hoc postulate, contrarily to what believed by the author of ref. [18].

At this point, we can eventually define the *mean value* of

.the time at which our "particle" passes through position x, travelling in the positive or negative direction of the x axis, respectively, as:

$$\langle t_+(x)\rangle \equiv \frac{\int_{-\infty}^{\infty} t J_+(x,t)dt}{\int_{-\infty}^{\infty} J_+(x,t)dt} \tag{14.a}$$

$$\langle t_-(x)\rangle \equiv \frac{\int_{-\infty}^{\infty} t J_-(x,t)dt}{\int_{-\infty}^{\infty} J_-(x,t)dt} \tag{14.a}$$

and, moreover, the variances of the distributions of these times as:

$$Dt_+(x) \equiv \frac{\int_{-\infty}^{\infty} t^2 J_+(x,t)dt}{\int_{-\infty}^{\infty} J_+(x,t)dt} - [\langle t_+(x)\rangle]^2 \tag{15.a}$$

$$Dt_-(x) \equiv \frac{\int_{-\infty}^{\infty} t^2 J_-(x,t)dt}{\int_{-\infty}^{\infty} J_-(x,t)dt} - [\langle t_-(x)\rangle]^2 \tag{15.b}$$

in accordance with the proposal presented in refs. [1,15].

Thus, we have a formalism for defining mean values, variances (and other central moments) related to the duration *distributions* of all possible processes for a particle, tunnelling through a potential barrier located in the interval $(0, a)$ along the x

axis; and not only for tunnelling, but also for all possible kinds of collisions, with arbitrary energies and potentials. For instance, we have that

$$\langle \tau_T(x_i, x_f) \rangle \equiv \langle t_+(x_f) \rangle - \langle t_+(x_i) \rangle \tag{16}$$

with $-\infty < x_i < 0$ and $a < x_f < \infty$; and therefore (as anticipated in eq. (3)) that

$$D\tau_T(x_i, x_i, x_f) \equiv Dt_+(x_f) + Dt_+(x_i),$$

for *transmissions* from region $(-x, 0)$ to region (a, ∞) which we called [1] regions I and III, respectively. Analogously, for the pure (complete) tunnelling process one has:

$$\langle \tau_{Tun}(0, a) \rangle \equiv \langle t_+(a) \rangle - \langle t_+(0) \rangle \tag{17}$$

and

$$D\tau_{Tun}(0, a) \equiv Dt_+(a) + Dt_+(0); \tag{18}$$

while one has

$$\langle \tau_{Pen}(0, x_f) \rangle \equiv \langle t_+(x_f) \rangle - \langle t_+(0) \rangle \tag{19}$$

and

$$D\tau_{Pen}(0, x_f) \equiv Dt_+(x_f) + Dt_+(0) \tag{20}$$

[with $0 < x_f < a$] for *penetration* inside the barrier region (which we called region II). Moreover:

$$\langle \tau_{Ret}(x, x) \rangle \equiv \langle t_-(x) \rangle - \langle t_+(x) \rangle \tag{21}$$

$$D\tau_{Ret}(x, x) \equiv Dt_-(x) + > Dt_+(x)\rangle \tag{22}$$

[with $0 < x < a0$] for "*return processes*" inside the barrier. At last, for *reflections* in region I, we have that:

$$\langle \tau_R(x_i, x_i) \rangle \equiv \langle t_-(x_i) \rangle - \langle t_+(x_i) \rangle \tag{23}$$

[with $-\infty < x_i < a$], and (as anticipated in eq. (4)) that $D\tau_R(x_i, x_i) \equiv Dt_-(x_i) + Dt_+(x_i)$.

Let us stress that our definitions hold within the framework of conventional quantum mechanics, without the introduction of any new postulates, and with the single measure expressed by weights (13.a), (13.b) for all time averages (both in the initial and in the final conditions).

According to our definition, the tunnelling phase time (or, rather, the transmission duration), defined by the stationary phase approximation for quasi-monochromatic particles, is meaningful *only* in the limit $x_i \to \infty$ when $J_+(x,t)$ is the flux density of the initial packet J_{in} of *incoming* waves (in absence of *reflected* waves).

Analogously, the dwell time, which can be represented (cf. eqs. (1), (2)) by the expression [7,8,12]

$$\bar{\tau}^{Dw}(x_i, x_f) = \left(\int_{-\infty}^{\infty} dt\, t\, J(x_f, t) - \int_{-\infty}^{\infty} dt\, t\, J(x_i) \right) \left(\int_{-\infty}^{\infty} dt\, J_{in}(x_i, t) \right)^{-1},$$

with $-\infty < x_i < 0$, and $x_f > a$, is not self consistent, generally speaking. In fact, the weight in the time averages is meaningful, positive defined and normalized to 1 *only* in the rare cases when $x_i \to -\infty$ and $J_{in} = J_{III}$ (i.e., when the barrier is transparent).

3. Penetration and Return process durations, inside a rectangular barrier, for tunnelling gaussian wave packets: Numerical results.

Some preliminary calculations of mean durations for penetration processes, inside a rectangular barrier, were presented by us in ref. [12] for tunnelling gaussian wave packets. Later on – looking for any possible explanations for the disagreement between the results in ref. [8] and in our ref. [12] – we discovered, however, that an exponential factor was missing in a term of one of the fundamental formulae on which the numerical computations (performed by one of us at Kiev) were based: a mistake that could not be detected, of course, by our careful checks about the computing process. We put forth here our new results, for a wider set of penetration and return processes, one of the aim being to check the *tunnelling speeds*. In our calculations, the initial wave packet is

$$\psi_{in}(x,t) = \int_0^{\infty} G(k-\bar{k}) \exp[ikx - iEt/\hbar] dk \tag{24}$$

with

$$G(k-\bar{k}) \equiv C \exp[-(k-\bar{k})^2/(2\Delta k)^2], \tag{25}$$

exactly as in ref. [8]; and with $E = \hbar^2 k^2/2m$; quantity C being the normalization constant, and m the electron mass. Our procedure of integration was described in ref. [12]. Some of the new numerical results appear in Figs. 1-2, which should replace Figs. 1-3 of ref. [12]. As anticipated above, by using the same parameters as (or parameters very near to) the ones adopted by Leavens for his Figs. 3 and 4 in ref. [8], our new figures result to be more similar to Leavens' than the uncorrected ones. One verifies once more, however, that – contrarily to the claim in ref. [8] – our theory appears to yield *in those cases* non-negative results for $\langle \tau_{Pen}(x_f) \rangle$. Actually, our previous general conclusions do not seem to be affected by the mentioned mistake. In particular, the value of $\langle \tau_{Pen}(x_f) \rangle$ increases with increasing x_f, and tends to saturation for $x_f \to a$. We acknowledge, however, that the difference in the adopted integration ranges [$-\infty$ to $+\infty$ for us, and 0 to $+\infty$ for Leavens] does not play an important role, contrarily to our previous belief. [12], in explaining the discrepancy between our results and Leavens'. It *might* perhaps depend on the fact that the functions to be integrated do fluctuate heavily(2) (anyway, we did carefully check that our elementary integration step in the integration over dk was small enough in order to guarantee the stability of the numerical result, and, in particular, of their sign, for strongly oscillating functions in the integrand). More probably, that discrepancy depends on the fact that Leavens seem to have used wave packets different from ours, since their initial conditions are actually different in our and in Leavens' work, as claimed also in the very recent ref. [13].

Let us express the penetration depth in ångstroms, and the penetration time in seconds. In Fig. 1 we show the plots corresponding to $a = 5$ Å, for $\Delta k = 0.02$ and 0.01 Å^{-1}, respectively.

The penetration time $\langle \tau_{Pen} \rangle$ always tends to a *saturation* value.

In Fig. 2 we show, for the case $\Delta k = 0.01$ Å^{-1}1, the plot corresponding to $a = 10$ Å. It is interesting that $\langle \tau_{Pen} \rangle$ is practically the same (for the same Δk) for $a = 5$ and $a = 10$ Å, a result that confirm, let us repeat, the existence [1] of the so-called *Hartman effect*. [2] Let us add that, when varying the parameter Δk between 0.005 and 0.15 Å^{-1} and letting a to assume values even larger than 10 Å, analogous results have been always gotten. Similar calculations have been performed (with quite reasonable results) also for various energies $\bar{E}$ in the range 1 to 10 eV.(3)

(2) We can *only* say that we succeeded in reproducing results of the type put forth in ref. [8] by using larger steps; whilst the "*non-causal*" results disappeared – *in the considered cases* – when adopting small enough integration steps.

(3) For the interested reader, let us recall that, when integrating over dt, we used the interval $-10^{-13}s$ to $+10^{-13}s$ (symmetrical with respect to $t = 0$), very much larger than the temporal wave packet extension. [Recall that the extension in time of a wave packets is of the order of $1/(\bar{v}\Delta k) = (\Delta k\sqrt{2\bar{E}/m})^{-1} \simeq 10^{-16}s$]. Our "centroid" has been always $t_0 = 0$; $x_0 = 0$. For clarity's sake, let us underline again that in our approach the initial wave packet $\psi_{in}(x, t)$ is

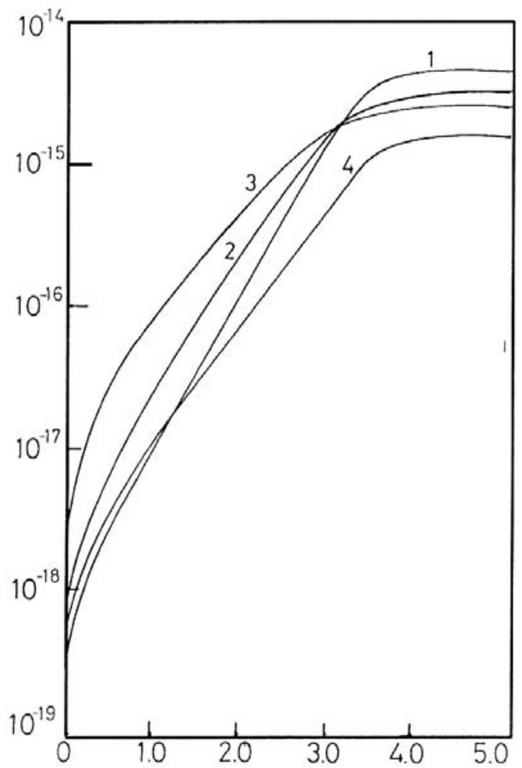

Fig. 3 - Behaviour of $\langle \tau_{Pen}(0, x) \rangle$ (expressed in seconds) as a function of x (expressed in ångstroms), for tunnelling through a rectangular barrier with width $a = 5$ Åand for different values of $\bar{E}$ and of Δk: curve 1: $\Delta k = 0.02$ Å^{-1} and $\bar{E} = 2.5$ eV; curve 2: $\Delta k = 0.02$ Å^{-1} and $\bar{E} = 5.0$ eV; curve 3: $\Delta k = 0.02$ Å^{-1} and $\bar{E} = 7.5$ eV; curve 4: $\Delta k = 0.04$ Å^{-1} and $\bar{E} = 5.0$ eV.

In Figs. 3, 4 and 5 we show the behaviour of the mean penetration and return durations as function of the penetration depth (with $x_i = 0$ and $0 \leq x_f \leq a$), for barriers with height

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not regarded as prepared at a certain instant of time, but it is expected to flow through any (initial) point x_i during the infinite time interval $(-\infty, +\infty)$, even if with a *finite* time-centroid t_0. The value of such centroid t_0 is essentially defined by the phase of the weight amplitude $G(k - \bar{k})$, and in our case is equal to 0 when $G(k - \bar{k})$ is real.

$V_0 = 10$ eV and width $a = 5$ Åor 10 Å. In Fig. 3 we present the plots of $\langle\tau_{Pen}(0, x)\rangle$ corresponding to different values of the mean kinetic energy: $\bar{E} = 2.5$ eV, 5 eV and 7.5 eV (plots 1, 2 and 3, respectively) with $\Delta k = 0.02$Å: and $\bar{E} = 5$ eV with $\Delta k = 0.04\text{Å}^{-1}$ (plot 4), always with $a = 5$Å. In Fig. 4 we show the plots of $\langle\tau_{Pen}(0, x)\rangle$, corresponding to $a = 5$Å, with $\Delta k = 0.02\text{Å}^{-1}$ and 0.04 Å^{-1} (plots 1 and 2, respectively); and to $a = 10$Å, with $\Delta k = 0.02\text{Å}^{-1}$ and 0.04 Å^{-1} (plots 3 and 4, respectively), the mean kinetic energy $\bar{E}$ being 5 eV, i.e., one half of V_0. In Fig. 5 the plots are shown of $\tau_{Ret}(x, x)$. The curves 1, 2 and 3 correspond to $\bar{E} = 2.5$ eV, 5 eV and 7.5 eV, respectively, for $\Delta k = 0.02\text{Å}^{-1}$ and $a = 5$Å; the curves 4, 5 and 6 correspond to $\bar{E} = 2.5$ eV, 5 eV and 7.5 eV, respectively, for $\Delta k = 0.04\text{Å}^{-1}$ and $a = 5$Å; while the curves 7, 8 and 9 correspond to $\Delta k = 0.02\text{Å}^{-1}$ and 0.04 Å^{-1}, respectively, for $\bar{E} = 5$ eV and $a = 10$Å.

Also from the new Figs. 3-5 one can see that: 1) at variance with ref. [8], no plot considered by us for the mean penetration duration $\langle\tau_{Pen}(0, x)\rangle$ of our wave packets presents any interval with negative values, nor with a decreasing $\langle\tau_{Pen}(0, x)\rangle$ for increasing x; and, moreover, that 2) the mean tunnelling duration $\langle\tau_{Tun}(0, a)\rangle$ does not depend on the barrier width a ("Hartman effect"); and finally that 3) quantity $\langle\tau_{Tun}(0, a)\rangle$ decreases when the energy increases. Furthermore, it is noticeable that also from Figs. 3-5 we observe: 4) a rapid increase for the value of the electron penetration time in the initial part of the barrier region (near $x = 0$); and 5) a tendency of $\langle\tau_{Pen}(0, x)\rangle$ to a saturation value in the final part of the barrier, near $x = a$.

Feature 2), firstly observed for quasi-monochromatic particles, [2] does evidently agree with the predictions made in ref. [1] for arbitrary wave packets. Feature 3) is also in agreement with previous evaluations performed for quasi-monochromatic particles and presented, for instance, in refs. [1,2,15]. Features

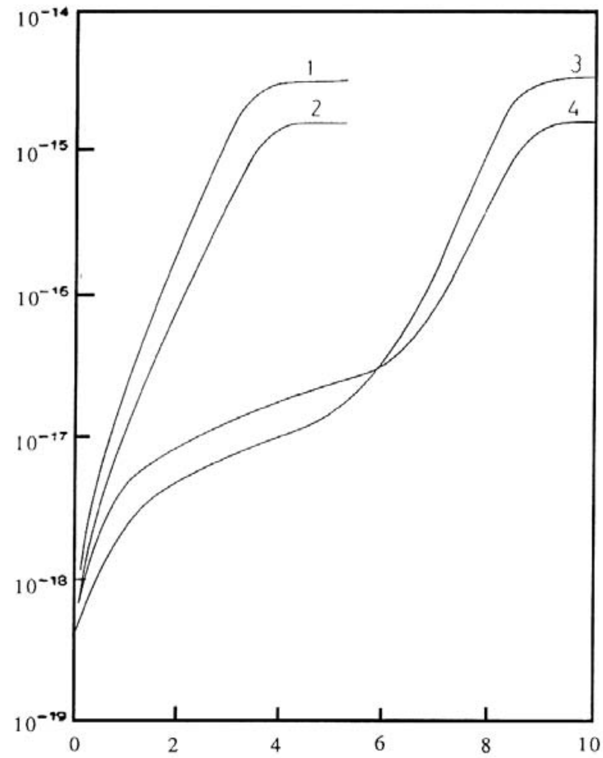

Fig. 4 - Behaviour of $\langle \tau_{Pen}(0, x) \rangle$ (in seconds) as a function of x (in ångstroms) for $\bar{E} = 5$ eV and different values of a and AΔk: curve 1: $a = 5$ Åand $\Delta k = 0.02$ Å^{-1}; curve 2: $a = 5$ Åand $\Delta k = 0.04$ Å^{-1}; curve 3: $a = 10$ Åand $\Delta k = 0.02$ Å^{-1}; curve 4: $a = 10$ Åand $\Delta k = 0.04$ Å^{-1}.

4) and 5) can be apparently explained by interference between those initial penetrating and returning waves inside the barrier, whose superposition yields the resulting fluxes J_+ and J_-. In particular, if in the initial part of the barrier the returning-wave packet is comparatively large, it does essentially extinguish the leading edge of the incoming-wave packet. By contrast, if for growing x the returning-wave packet quickly vanishes, then the contribution of the leading edge of the incoming-wave packet to the mean penetration duration $\langle \tau_{Pen}(0, x) \rangle$ does initially (quickly)

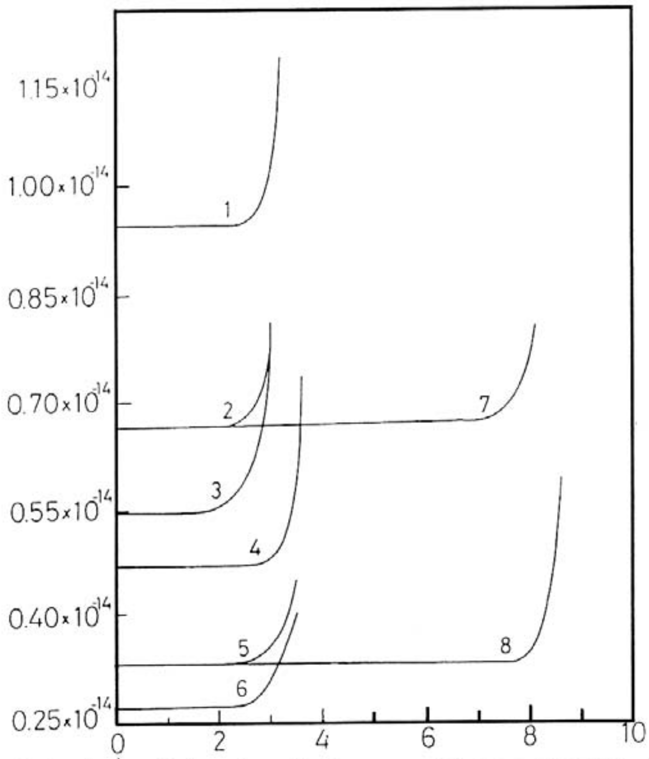

Fig. 5 - Behaviour of $\langle\tau_{Ret}(x,x)\rangle$ (in seconds) as a function of x (in ångstroms) for different values of a, $\bar{E}$ and Δk: curve 1: $a = 5$ Å, $\bar{E} = 2.5$ eV and $\Delta k = 0.02$ Å^{-1}; curve 2: $a = 5$ Å, $\bar{E} = 5.0$ eV and $\Delta k = 0.02$ Å^{-1}; curve 3: $a = 5$ Å, $\bar{E} = 7.5$ eV and $\Delta k = 0.02$ Å^{-1}; curve 4: $a = 5$ Å, $\bar{E} = 2.5$ eV and $\Delta k = \neg 0.02$ Å^{-1}; curve 5: $a = 5$ Å, $\bar{E} = 5.0$ eV and $\Delta k = 0.02$ Å^{-1}; curve 6: $a = 5$ Å, $\bar{E} = 7.5$ eV and $\Delta k = 0.02$ Å^{-1}; curve 7: $a = 5$ Å, $\bar{E} = 5.0$ eV and $\Delta k = 0.02$ Å^{-1}; curve 8: $a = 5$ Å, $\bar{E} = 5.0$ eV and $\Delta k = 0.02$ Å^{-1}.

grow, while in the final barrier region its increase does rapidly slow down.

Furthermore, the larger is the barrier width a, the larger is the part of the back edge of the incoming-wave packet which is extinguished by interference with the returning-wave packet. Quantitatively, these phenomena will be studied elsewhere. Finally, in connection with the plots of $\langle\tau_{Ret}(x,x)\rangle$ as a function of x, presented in Fig. 5, let us observe that: (i) the mean reflection duration $\langle\tau_R(0,0)\rangle \equiv \langle\tau_{Ret}(0,0)\rangle$ does not depend on the barrier width a; (ii) in correspondence with the barrier region betwen 0 and approximately 0.6 a, the value of $\langle\tau_{Ret}(0,x)\rangle$ is almost constant; while (iii) it increases with x only in the barrier region near $x = a$ (even if it should be pointed out that our calculations near $x = a$ are not so good, due to the very small values assumed by $\int_{-\infty}^{\infty} J_-(x,t)dt$ therein). Let us notice that point (i), also observed firstly for quasi-monochromatic particles, [2] is as well in accordance with the results obtained in ref. [1] for arbitrary wave packets. Moreover, also points (ii) and (iii) can be explained by interference phenomena inside the barrier: if, near $x = a$, the initial returning-wave packet is almost totally quenched by the initial incoming-wave packet, then only a negligibly small piece of its back edge (consisting in the components with the smallest velocities) does remain. With decreasing x $(x \rightarrow 0)$, the unquenched part of the returning-wave packet seems to become more and more larger (containing more and more large velocity components), thus making the difference $\langle\tau_{REt}(0,x)\rangle - \langle\tau_{Pen}(0,x)\rangle$ almost constant. And the interference between incoming and reflected waves at points $x \leq< 0$ does effectively constitute a retarding phenomenon [so that $\langle t_-(x=0)\rangle$ is larger than $\langle\tau_R(x=0)\rangle$], which can explain the larger values of $\langle\tau_R(x=0, x=0)\rangle$ in comparison with $\langle\tau_{Tun}(x=0, x=a)\rangle$.

Therefore our formulae, in all the cases considered above, did not forward any negative values for the calculated times, confirming our previous analysis (and conclusions) at page 352 in ref. [1], concerning in particular the validity of the Hartman effect also for *non*-quasi-monochromatic wave packets. Even more, since the interference between incoming and reflected waves before the barrier (or between penetrating and returning waves, inside the barrier, near the entrance wall) does just *increase* the tunnelling time as well as the transmission times, we can expect that our non-relativistic formulae for $\langle\tau_{Tun}(0, a)\rangle$ and $\langle\tau_T(x_i\langle 0, x_f\rangle a)\rangle$ will always forward positive values.(4)

At this point, it is essential – however – to observe the following. Even if our non-relativistic equations are not expected (as we have seen above) to yield negative times, nevertheless one ought to bear in mind that (whenever it is met an object, $\mathcal{O}$, travelling at Superluminal speed) negative contributions should be expected to the tunnelling times: and this ought not to be regarded as unphysical. In fact, whenever the "object" $\mathcal{O}$ *overcomes* the infinite speed [19] with respect to a certain observer, it will afterwards appear to the same observer as the anti-object $\bar{\mathcal{O}}$ travelling in the opposite *space* direction [19]. For instance, when passing from the lab to a frame $\mathcal{F}$ moving in the em same direction as the particles or waves entering the barrier region, the "objects" penetrating through the final part of the barrier (with almost infinite speeds, like in Figs. 1-5) will appear in the frame $\mathcal{F}$ as "anti-objects" crossing that portion of the barrier in the opposite space-direction...[19]. In the new frame $\mathcal{F}$. therefore, such "anti-objects" $\bar{\mathcal{O}}$ would yield a *negative* contribution to the tunnelling time: which could even result, in total, to be negative. For any clarifications, see refs. [19]. So, we have no objections a priori against the fact that Leavens can find, in certain cases, negative values [8,13]: e.g, when applying our

(4) A different claim by Delgado, Brouard and Muga [13] does not seem to be relevant to our calculations, since it is based once more, like ref. [8], not on our but on different wave packets (and over-barier components are also retained in ref. [13], at variance with us). Moreover, in their classical example, they overlook the fact that the mean entrance time $\langle t_+(0)\rangle$ gets contribution mainly by the rapid components of the wave packet; the slow components are (almost) totally reflected by the initial wall, causing a quantum-mechanical reshaping that contributes to the initial "time decrease" discussed by us already in the last few paragraphs of page 352 in ref. [1]. All such phenomena *reduce* the value of $\langle t_+(0)\rangle$, and we expect it to be (in our non-relativistic treatment) less than $\langle t_+(a)\rangle$.

formulae to wave packets with suitable initial conditions. What we want to stress here is that the appearance of negative times (it being predicted by Relativity itself, [19] when in presence of something travelling faster than c) is not a valid reason to rule out a theoretical approach.

4. Further remarks.

In ref. [8] it has been critically commented also on our view about performing actual averages over the physical time. We cannot agree with those comments. Let us re-emphasize that, within conventional quantum mechanics, the time $t(x)$ at which our particle (wave packet) passes through the position x is "statistically distributed" with the probability densities

$$dt\, J_{\pm}(x,t) / \int_{-\infty}^{\infty} dt\, J_{\pm}(x,t),$$

as we explained at page 350 of ref. [1]. This distribution meets the requirements of the time-energy uncertainty relation.

The last object of the criticism in ref. [8] refers to the impossibility, in our approach, of distinguishing between "to be transmitted" and "to be reflected" wave packets at the leading edge of the barrier. Actually, we do *distinguish* between them; only, we cannot – of cours – *separate* them, due to the obvious superposition (and interference) of both wave functions in $\rho(x,t)$, in $J(x,t)$ and even in $J \pm (x,t)$. This is known to be an unavoidable consequence of the superposition principle, valid for *wave functions* in conventional quantum mechanics. That last objection, therefore, should be addressed to quantum mechanics, rather then to us. Nevertheless, Leavens' aim of comparing the definitions proposed by us for the tunnelling times not only with conventional, but also with non-standard quantum mechanics *might* be regarded a priori as stimulating and possibly worth of further investigation.

Now, let us here explicitly notice that, when separating J_+ and J_-, we do *not totally* exclude superposition and interference between incoming and reflected waves: we do only (automatically) get a prevalence of incoming or reflected waves in J_+ and J_-, respectively.

The situation with causality for relativistic tunnelling particles is more complicated. The Hartman-Fletcher phenomenon (very small tunnelling durations), with the consequence of Superluminal velocities for sufficiently wide barriers, was confirmed theoretically also in QFT for Klein-Gordon and Dirac equations, [1] and experimentally for electromagnetic evanescent-mode wave packets [4-6] (tunnelling photons). It should be stressed that the problem of Superluminal velocities for electromagnetic wave packets in media with anomalous dispersion, with absorption, or behaving as a barrier for photons (such as regions with frustrated internal reflection) has been present in the scientific literature since long (see, for instance, quotations [2,1,19], and refs. therein). But the a complete settlement of the causal problem for relativistic waves (differently from the case of point particles [19]) is not yet available. Apparently, it is not sufficient to pay attention only to group velocity and mean duration for our particle to pass through a medium; by contrast, it is important ab initio taking into account and studying the *variances* (and the higher order central moments) of the duration distributions, as well as the wave packet *reshaping* in presence of a barrier, or inside anomalous media (even if, often, it does not play an essential role).

As to the approaches *alternative* to the direct description of tunnelling processes in terms of wave packets, let us here recall those ones which are based on averaging over the set of all dynamical paths (through the Feynman path integral formulation, the Wigner distribution method, and the non-conventional Bohm approach), and others that use additional degree of freedom which can be used as "clocks". A general analysis of all such alternative approaches can be found in refs. [1, 20-24], from different points of view.

If one confines himself within the framework of conventional quantum mechanics, then the Feynman path integral formulation seems to be adequate. [24] But it is not clear what procedure is needed to calculate physical quantities within the Feynman-type approach [23], and usually such calculations result in complex tunnelling durations. The Feynman approach seems to need further modifications if one wants to apply it to the time analysis of tunnelling processes, and its results obtaind up to now cannot be considered as final.

At last, as to the approaches based on introducing additional degrees of freedom as "clocks", one can often realize that the tunnelling time happens to be noticeably distorted by the presence of those degrees of freedom. For example, the Büttiker-Landauer time is connected with absorption or emission of modulation quanta (caused by the time-dependent oscillating part of the barrier potential) during tunnelling, rather than with the tunnelling process itself. [1,15] And, in connection with the Larmor precession time, it was shown [11,20] that such a definition is connected not only with the intrinsic tunnelling process. but also with the geometric boundaries of the magnetic field introduced as a part of the clock: for instance, if the magnetic field region is infinite, one actually ends up with the phase tunnelling time, after an average over the (small) energy spread of the wave packet. Anyway, those "clock" approaches, when applied to tunnelling wave packets, seem to lead – after having eliminated the distortion caused by the additional degrees of freedom – to the same results as the direct wave packet approach, whatever be the weight function adopted in the time integration.

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